Castellano and Pastor-Satorras Reply: The Comment by Ha *et al.* [1] criticizes our recent result [2] that the contact process (CP) on uncorrelated scale-free (SF) networks does not behave according to heterogeneous mean-field (MF) theory. The criticism is based on the following three claims: (1) The relative density fluctuations discussed in Fig. 4 of [2] are well reproduced by a Gaussian ansatz, Fig. 1 of [1]. (2) A numerical estimate of the density decay for $\gamma = 2.75$ agrees with the MF prediction $\theta = 1/(\gamma-2)$. (3) An estimate of the finite-size scaling (FSS) exponent $\alpha = \beta/\nu_\perp = 0.59(2)$ for the same γ agrees with the MF conjecture $\alpha = 1/(\gamma-1)$. We reply to these three points in the following paragraphs.

(1) The scaling form shown in Fig. 1 of [1] seems rather interesting. However, it does not affect the main conclusions of our work. In fact, it proves that for almost all values of k, the unscaled relative density fluctuations r_k are larger than 1 and the height of the maximum diverges with the network size. A diverging ratio r_k contradicts the MF assumption that the densities ρ_k are well defined quantities, and therefore naturally hints more towards failure of MF theory. Concerning the numerical points (2) and (3), we note that the critical point $p_c = 0.4240$ quoted in [1] is not compatible with our own estimate $p_c = 0.4215(5)$ [2], hinting that a diffent network model is being used in [1]. In any case, the estimate of θ extracted from Fig. 2 in [1] is utterly implausible: the value is arbitrarily obtained from an extremely short and noisy "plateau", whose existence appears more as the effect of chance than a real physical feature. Concerning point (3), we are puzzled by the conjecture $\alpha = 1/(\gamma-1)$, of which no proof is given, except a mention to "the hyperscaling argument" [1]. We note, however, that it is possible to obtain it from a trivial, although misleading, argument: in Ref. [2] we observed that the MF density of surviving particles at criticality should scale in uncorrelated SF networks as $\bar{\rho}_a \sim k_c^{-1}$, where k_c is the degree cut-off. Assuming that $k_c \sim N^{1/2}$, corresponding to *real uncorrelated* networks [3], our original result $\alpha = 1/2$ is recovered. On the other hand, taking $k_c \sim N^{1/(\gamma-1)}$, which corresponds to *real correlated* networks, yields the value $\alpha = 1/(\gamma-1)$. This result, however, makes little sense, since the MF theory developed in our paper [2] (and presumably in [1]) deals with *uncorrelated* networks.

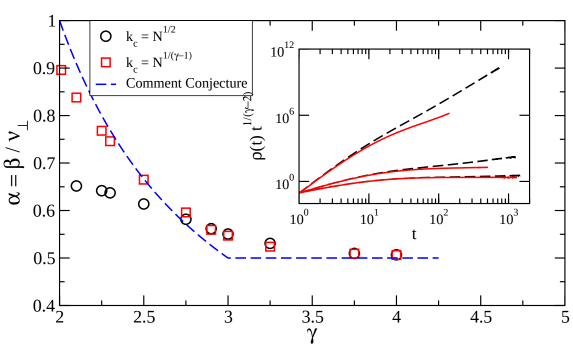

FIG. 1: Exponent $\alpha = \beta/\nu_\perp$ for RN SF networks for both cut-off scalings, compared with the conjecture $\alpha = 1/(\gamma-1)$. Inset: Check of the scaling $\rho(t) \sim t^{-1/(\gamma-2)}$, for $k_c \sim N^{1/2}$ (dashed lines) and $k_c \sim N^{1/(\gamma-1)}$ (solid lines). Values of γ (bottom to top): 2.75, 2.50 and 2.20.

In order to shed light on the true MF behavior of the model, we have performed new simulations of the CP on the random neighbors (RN) version of SF networks, in which p_c takes the exact MF value 1/2 [2], considering the two different cut-off scalings discussed above. In the main plot of Fig. 1 we show results corresponding to the estimate of the α exponent at p_c. We observe that the results for the realistic cut-off $k_c \sim N^{1/2}$ deviate strongly from the conjecture $\alpha = 1/(\gamma-1)$, which is not even correct for the unphysical cut-off (in uncorrelated networks) $k_c \sim N^{1/(\gamma-1)}$. Finally, we consider a more natural way to check the value of the θ exponent: since $\rho(t) \sim t^{-\theta}$, the function $\rho(t)t^\theta$ should show a plateau at intermediate values of t for the correct value of θ. In the inset of Fig. 1 the MF value $\theta = 1/(\gamma-2)$ is tested, for both cut-offs considered. As we can see, the MF value is reasonable for large γ, but it fails completely for γ close to 2.

From this numerical evidence we conclude that the MF conjecture presented in the Comment is at best only approximately valid for the unphysical case of uncorrelated networks with cut-off $k_c \sim N^{1/(\gamma-1)}$, which can only be constructed in the RN version of SF networks, while the MF prediction for θ fails completely for small γ and any k_c. Therefore, the main conclusion of paper [2], the invalidity of MF theory for *real uncorrelated* SF networks, remains unchallenged.

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